

9.3 Heat Conduction in a Square

This simple 2-D problem is used to illustrate input file construction and basic output. Heat conduction in a 1-meter square with an initial temperature, $T_0 = 200^\circ\text{C}$, is modeled after a surface temperature, $T_s = 100^\circ\text{C}$, is imposed at time, $t = 0$ (Fig. 5). The input parameters used for the heat-conduction problem are defined in Table VIII. The finite-element mesh for this problem is shown in Fig. 6. Only a quarter of the square needs to be modeled because of problem symmetry.

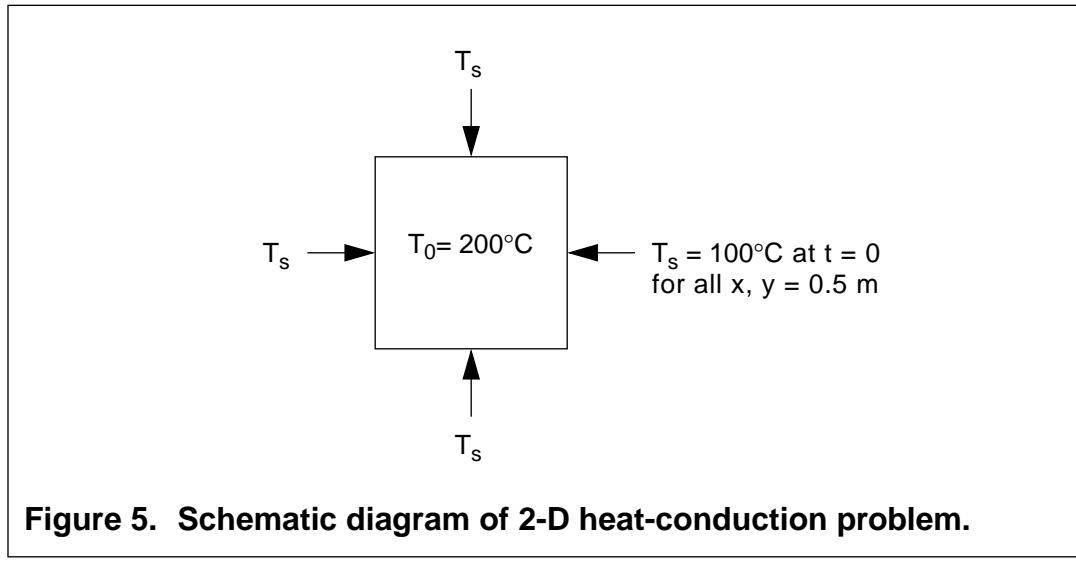


Table VIII. Input parameters for the 2-D heat-conduction problem

Parameter	Symbol	Value
Rock thermal conductivity	κ_r	$2.7 \frac{\text{W}}{\text{m} \cdot \text{K}}$
Rock density	ρ_r	2700 kg/m^3
Rock specific heat	C_r	$1000 \frac{\text{J}}{\text{kg} \cdot \text{K}}$
Width	a	0.5 m
Length	b	0.5 m
Initial temperature	T_0	200°C
Surface temperature for all $x, y = 0.5 \text{ m}$	T_s	100°C
Rock thermal diffusivity	$\kappa = \frac{\kappa_r}{\rho_r C_r}$	

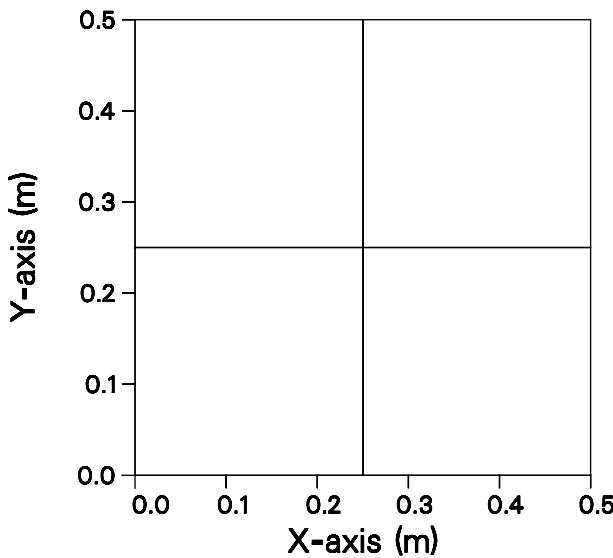


Figure 6. Finite-element mesh used for 2-D heat-conduction problem.

The input file (see Fig. 7) uses the optional macro control statement **node** (output nodes) and the required macro control statements **sol** (solution specification - heat transfer only), **init** (initial value data), **rock** (rock properties), **cond** (thermal conductivities), **perm** (permeabilities), **time** (simulation timing data), **ctrl** (program control parameters), **coor** (node coordinates), **elem** (element node data), and **stop**. For this problem, the macro control statement **flow** is also used to set the temperature boundary conditions. A portion of the output file is reproduced in Fig. 8.

The analytical solution for 2-D heat conduction (Carslaw and Jaeger, 1959) is given by

$$T = T_s + \frac{16(T_0 - T_s)}{\pi^2} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \frac{(-1)^{m+n}}{(2m+1)(2n+1)} \cos \frac{(2m+1)\pi x}{2a} \cos \frac{(2n+1)\pi y}{2b} e^{-\alpha_{m,n} t}$$

where $\alpha_{m,n} = \frac{\kappa\pi^2}{4} \left[\frac{(2m+1)^2}{a^2} + \frac{(2n+1)^2}{b^2} \right]$ and the region is taken to be

$$-a < x < a, -b < y < b.$$

Figure 9 shows a plot of the simulation results compared to the analytical solution for the selected output nodes.

```
***** 2-D Heat Conduction Model (2X2 rectangles) *****
node
 2
 7  5
sol
 -1 -1
init
 10. 0. 200. 0. 0. 200. 0. 0.
rock
 1  9  1  2700. 1000. 0.

cond
 1  9  1  2.7e-00 2.7e-00 2.7e-00

perm
 1  9  1  1.e-30 1.e-30 1.e-30

flow
 1  3  1  10.00 -100.00 1.e03
 3  9  3  10.00 -100.00 1.e03

time
 0.005 4.00 1000 10 1994 02

ctrl
 40  1.e-04 08
 1  9  1  1

 1.0  0.0  1.0
 10  1.0  0.00005 0.005
 1  0

coor Feb 23, 1994 11:39:40
 9
 1  0.  0.50  0.
 2  0.25  0.50  0.
 3  0.50  0.50  0.
 4  0.  0.25  0.
 5  0.25  0.25  0.
 6  0.50  0.25  0.
 7  0.  0.  0.
 8  0.25  0.  0.
 9  0.50  0.  0.

elem
 4  4
 1  4  5  2  1
 2  5  6  3  2
 3  7  8  5  4
 4  8  9  6  5

stop
```

Figure 7. FEHM input file for heat-conduction example (heat2d.in).

FEHM 01.00 [sun4] 96/07/24 10:46:53

File purpose - Variable - Unit number - File name

```

control      - ioctl - 0 - not using
input        - inpt  - 11 - heat2d.in
geometry     - incoor - 11 - heat2d.in
zone         - inzone - 11 - heat2d.in
output        - iout  - 14 - heat2d.out
initial state - iread - 0 - not using
final state   - isave  - 16 - heat2d.fin
time history  - ishis  - 17 - heat2d.his
time his.(tr) - istrc - 0 - not using
contour plot - iscon - 0 - not using
con plot (dp) - iscon1 - 0 - not using
fe coef stor - isstor - 0 - not using
input check   - ischk  - 22 - heat2d.chk

```

Value provided to subroutine user: not using

***** 2-D Heat Conduction Model (2X2 rectangles) *****

```

**** input title : coor **** incoor = 11 ****
**** input title : elem **** incoor = 11 ****
**** input title : stop **** incoor = 11 ****
**** input title : node **** inpt = 11 ****
**** input title : sol **** inpt = 11 ****
**** input title : init **** inpt = 11 ****
**** input title : rock **** inpt = 11 ****
**** input title : cond **** inpt = 11 ****
**** input title : perm **** inpt = 11 ****
**** input title : flow **** inpt = 11 ****
**** input title : time **** inpt = 11 ****
**** input title : ctrl **** inpt = 11 ****
**** input title : stop **** inpt = 11 ****
BC to BC connection(s) found(now set=0.0)

```

Figure 8. FEHM output from the 2-D heat-conduction example .

pressures and temperatures set by gradients

storage needed for ncon	43 available	43
storage needed for nop	43 available	46
storage needed for a matrix	33 available	33
storage needed for b matrix	33 available	46
storage needed for gmres	81 available	81
storage available for b matrix resized to	33<<<<	

time for reading input, forming coefficients 0.333E-01

**** analysis of input data on file fehmn.chk ****

Time Step 1

Timing Information

Years	Days	Step Size (Days)
0.136893E-04	0.500000E-02	0.500000E-02

Cpu Sec for Time Step = 0.1667E-01 Current Total = 0.1667E-01

Equation Performance

Number of N-R Iterations: 1

Avg # of Linear Equation Solver Iterations: 3.0

Number of Active Nodes: 9.

Total Number of Newton-Raphson Iterations: 1

Node	Equation 1 Residual	Equation 2 Residual
------	---------------------	---------------------

7	0.111444E-07	0.185894E-01
---	--------------	--------------

5	0.165983E-07	0.135450E+01
---	--------------	--------------

Nodal Information (Water)

	source/sink	source/sink
--	-------------	-------------

Node	p(MPa)	e(MJ)	l sat	temp(c)	(kg/s)	(MJ/s)
------	--------	-------	-------	---------	--------	--------

7	10.000	0.00	0.000	199.981	0.	0.
---	--------	------	-------	---------	----	----

5	10.000	0.00	0.000	198.645	0.	0.
---	--------	------	-------	---------	----	----

Global Mass & Energy Balances

Total mass in system at this time: 0.000000E+00 kg

Total mass of steam in system at this time: 0.000000E+00 kg

Total enthalpy in system at this time: 0.105123E+03 MJ

Water discharge this time step: 0.000000E+00 kg (0.000000E+00 kg/s)

Water input this time step: 0.000000E+00 kg (0.000000E+00 kg/s)

Total water discharge: 0.000000E+00 kg (0.000000E+00 kg/s)

Total water input: 0.000000E+00 kg (0.000000E+00 kg/s)

Enthalpy discharge this time step: 0.297800E+02 MJ (0.689352E-01 MJ/s)

Enthalpy input this time step: 0.000000E+00 MJ (0.000000E+00 MJ/s)

Figure 8. FEHM output from the 2-D heat-conduction example (continued).

Total enthalpy discharge: 0.297800E+02 MJ (0.689352E-01 MJ/s)
 Total enthalpy input: 0.297800E+02 MJ (0.689352E-01 MJ/s)

Net kg water discharge (total out-total in): 0.000000E+00
 Net MJ discharge (total out-total in): 0.000000E+00
 Conservation Errors: 0.000000E+00 (mass), -0.100326E+01 (energy)

 Time Step 11

 Time Step 801

Timing Information

Years	Days	Step Size (Days)
0.109515E-01	0.400005E+01	0.500000E-04

Cpu Sec for Time Step = 0. Current Total = 4.533

Equation Performance

Number of N-R Iterations: 1
 Avg # of Linear Equation Solver Iterations: 2.0
 Number of Active Nodes: 9.
 Total Number of Newton-Raphson Iterations: 801
 Node Equation 1 Residual Equation 2 Residual
 7 0.977369E-13 0.186062E-04
 5 0.621566E-13 0.930309E-05

Nodal Information (Water)

Node	p(MPa)	e(MJ)	I sat	temp(c)	(kg/s)	(MJ/s)
7	10.000	0.00	0.000	100.230	0.	0.
5	10.000	0.00	0.000	100.115	0.	0.

Global Mass & Energy Balances

Total mass in system at this time: 0.000000E+00 kg
 Total mass of steam in system at this time: 0.000000E+00 kg
 Total enthalpy in system at this time: 0.675565E+02 MJ

Water discharge this time step: 0.000000E+00 kg (0.000000E+00 kg/s)
 Water input this time step: 0.000000E+00 kg (0.000000E+00 kg/s)
 Total water discharge: 0.000000E+00 kg (0.000000E+00 kg/s)
 Total water input: 0.000000E+00 kg (0.000000E+00 kg/s)

Enthalpy discharge this time step: 0.455636E-05 MJ (0.105471E-05 MJ/s)
 Enthalpy input this time step: 0.000000E+00 MJ (0.000000E+00 MJ/s)
 Total enthalpy discharge: 0.673463E+02 MJ (0.155894E+02 MJ/s)

Figure 8. FEHM output from the 2-D heat-conduction example (continued).

Total enthalpy input: 0.673463E+02 MJ (0.155894E+02 MJ/s)

Net kg water discharge (total out-total in): 0.000000E+00

Net MJ discharge (total out-total in): 0.000000E+00

Conservation Errors: 0.000000E+00 (mass), -0.100144E+01 (energy)

simulation ended: days 4.00 timesteps 801
total newton-raphson iterations = 801

total code time(timesteps) = 4.483334

****-----
**** This program for ****
**** Finite Element Heat and Mass Transfer in porous media ****
****-----
**** Version : FEHM 01.00 [sun4] ****
**** End Date : 96/07/24 ****
**** Time : 10:46:59 ****
****-----

Figure 8. FEHM output from the 2-D heat-conduction example (continued).

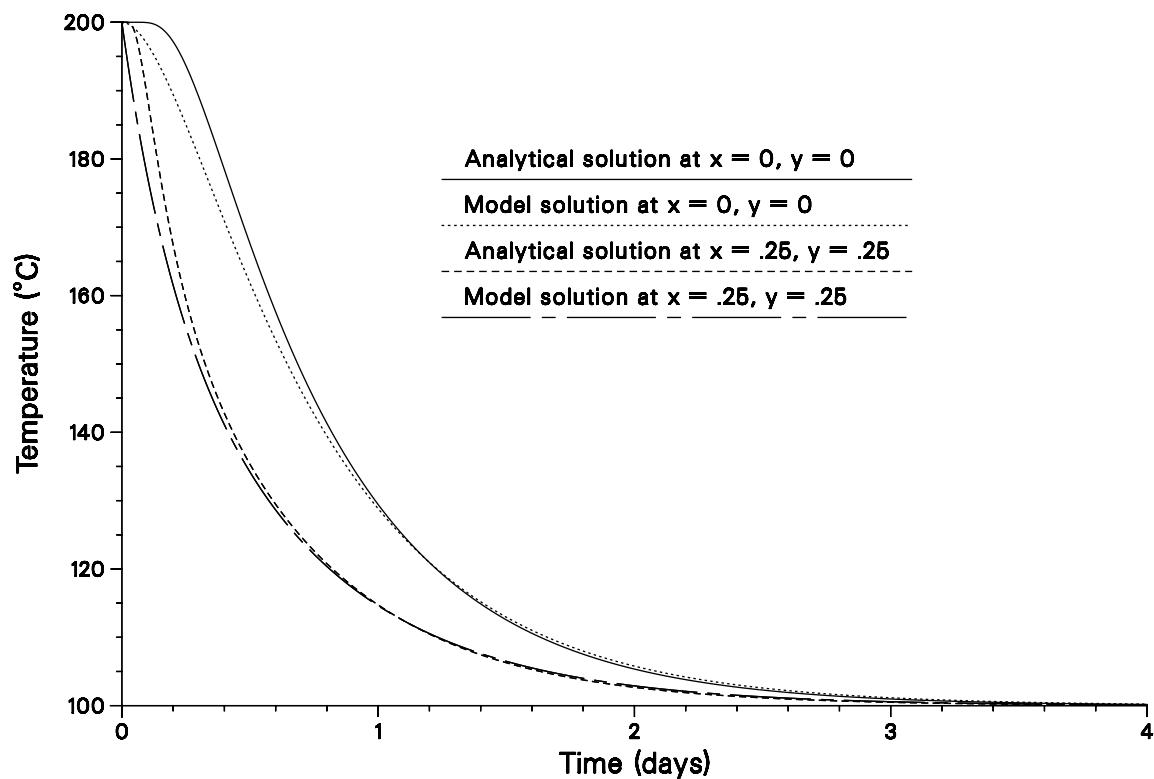


Figure 9. Comparison of analytical and model solution for 2-D heat conduction.